Analysis of the scalar doubly heavy tetraquark states with QCD sum rules

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Abstract

In this article, we perform a systematic study of the mass spectrum of the scalar doubly charmed and doubly bottom tetraquark states using the QCD sum rules.

1 Introduction

The $Z(4430)$ and the $Z(4050)$, $Z(4250)$ observed in the decay modes $\psi'\pi^+$ and $\chi_{c1}\pi^+$ respectively by the Belle collaboration are the most interesting subjects [1, 2, 3]. We can distinguish the multiquark states from the hybrids or charmonia with the criterion of non-zero charge. They can't be pure $c\bar{c}$ states due to the positive charge, and must be some special combinations of the $c\bar{c}u\bar{d}$ tetraquark states, irrespective of the molecule type and the diquark-antidiquark type. If those states are confirmed in the future, they are excellent candidates for the heavy tetraquark states of the $Qq\bar{Q}\bar{q}'$ type. It is interesting to explore the possibility that whether or not there exist doubly heavy tetraquark states of the $QQ\bar{q}\bar{q}'$ type.

On the other hand, the QCD sum rules is a powerful theoretical tool in studying the ground state hadrons [4, 5]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [4, 5].

There have been several successful applications of the QCD sum rules in studying the hidden charmed and hidden bottom tetraquark states ($Qq\bar{Q}\bar{q}'$ type). In Refs.[6, 7, 8], we study the mass spectrum of the scalar hidden charmed and hidden bottom tetraquark states in a systematic way using the QCD sum rules, and identify the $Z(4250)$ tentatively as a scalar tetraquark state of the diquark-antidiquark type; while in Ref.[9] the $Z(4050)$ and $Z(4250)$ are interpreted as the $D_1\bar{D}$ molecular state. In Refs.[10, 11], we study the mass spectrum of the vector hidden charmed and hidden bottom tetraquark states systematically using the QCD sum rules. In Ref.[12], we perform a systematic study of the mass spectrum of the axial-vector hidden charmed and hidden bottom tetraquark states using the QCD sum rules, and identify the $Z(4430)$ tentatively as an axial-vector tetraquark state of the diquark-antidiquark type. In Refs.[13, 14], Lee et al study the $Z(4430)$ with the QCD sum rules and observe that the $Z(4430)$ maybe a 0^- molecular type or diquark-antidiquark type tetraquark state. In Ref.[15], Chen et al study the 0^{--} hidden charmed and hidden bottom tetraquark states in details with the QCD sum rules.

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In Ref.[16], Navarra et al use the QCD sum rules to study the possible existence of the doubly heavy tetraquark states $QQ\bar{u}\bar{d}$ with $J^P = 1^+$. There have been several other theoretical approaches in studying the doubly heavy tetraquark states, such as the potential models and QCD inspired potential models [17, 18, 19], solving the four-body problem within a non-relativistic quark model [20, 21], the variational method combined with a non-relativistic potential model [22], the chiral constituent quark model [23, 24], the semi-empirical mass relations [25], the relativistic quark model based on a quasipotential approach in QCD [26], etc. Whether or not there exist the doubly charmed or doubly bottom tetraquark configurations is of great importance itself, because it provides a new opportunity for a deeper understanding of the low energy QCD.

It is interesting to study the mass spectrum of the doubly heavy tetraquark states ($QQ\bar{q}\bar{q}'$ type) with the QCD sum rules, and make an independent estimation from QCD. In Refs.[27, 28, 29], we study the mass spectrum of the $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ doubly heavy baryon states (QQq type) in a systematic way using the QCD sum rules. In this article, we extend our previous works to study the mass spectrum of the scalar doubly charmed and doubly bottom tetraquark states in a systematic way with the QCD sum rules.

We take the diquarks as the basic constituents following Jaffe and Wilczek [30, 31], and construct the doubly heavy tetraquark states with the diquark and antidiquark pairs. The diquarks have five Dirac tensor structures, scalar $C\gamma_5$, pseudoscalar C, vector $C\gamma_\mu\gamma_5$, axial-vector $C\gamma_\mu$ and tensor $C\sigma_{\mu\nu}$, where C is the charge conjunction matrix. The structures $C\gamma_\mu$ and $C\sigma_{\mu\nu}$ are symmetric while the structures $C\gamma_5$, C and $C\gamma_\mu\gamma_5$ are antisymmetric.

The scattering amplitude for one-gluon exchange in an $SU(N_c)$ gauge theory is proportional to

$$T_{ki}^a T_{lj}^a = -\frac{N_c+1}{4N_c}(\delta_{jk}\delta_{il} - \delta_{ik}\delta_{jl}) + \frac{N_c-1}{4N_c}(\delta_{jk}\delta_{il} + \delta_{ik}\delta_{jl}), \tag{1}$$

where the T^a is the generator of the gauge group, and the i, j and k, l are the color indexes of the two quarks in the incoming and outgoing channels respectively. For $N_c = 3$, the negative sign in front of the antisymmetric antitriplet indicates the interaction is attractive, while the positive sign in front of the symmetric sextet indicates the interaction is repulsive [32]. On the other hand, the scattering amplitude for one-gluon exchange in the Dirac spinor space is proportional to

$$(\gamma_\mu)_{ij}(\gamma^\mu)_{kl} = -(\gamma_5 C)_{ik}(C\gamma_5)_{lj} + (C)_{ik}(C)_{lj} + \frac{1}{2}(\gamma_5\gamma_\alpha C)_{ik}(C\gamma^\alpha\gamma_5)_{lj}$$
$$-\frac{1}{2}(\gamma_\alpha C)_{ik}(C\gamma^\alpha)_{lj}, \tag{2}$$

the negative sign in front of the scalar and axial-vector channels indicates the interaction is attractive.

For the doubly heavy quark system with the same flavor $Q^i C\Gamma Q^j$, where the Γ denote the Dirac matrixes 1, γ_5, γ_μ, $\gamma_\mu\gamma_5$ and $\sigma_{\mu\nu}$, the color indexes i and j should be antisymmetric, i.e.

$$Q^i C\Gamma Q^j \sim \epsilon_{ijk} Q^i C\Gamma Q^j. \tag{3}$$

In the case of the antisymmetric structures $C\gamma_5$, C and $C\gamma_\mu\gamma_5$, the fermi statistics forbids the formulation of the diquark states.

In this article, we use the symmetric structure $C\gamma_\mu$ to construct the interpolating currents $J(x)$ to study the doubly charmed and doubly bottom tetraquark states Z:

$$
\begin{aligned}
J_{qq}(x) &= \epsilon^{ijk}\epsilon^{imn}Q_j^T(x)C\gamma_\mu Q_k(x)\bar{q}_m(x)\gamma^\mu C\bar{q}_n^T(x)\,, \\
J_{qs}(x) &= \epsilon^{ijk}\epsilon^{imn}Q_j^T(x)C\gamma_\mu Q_k(x)\bar{q}_m(x)\gamma^\mu C\bar{s}_n^T(x)\,, \\
J_{ss}(x) &= \epsilon^{ijk}\epsilon^{imn}Q_j^T(x)C\gamma_\mu Q_k(x)\bar{s}_m(x)\gamma^\mu C\bar{s}_n^T(x)\,,
\end{aligned}
\tag{4}
$$

where $q = u, d$. In the isospin limit, the interpolating currents $J(x)$ result in three distinct expressions for the correlation functions $\Pi(p)$, which are characterized by the number of the s quark they contain. In Refs.[7, 11], we observe that the ground state masses of the scalar and vector hidden charmed and hidden bottom tetraquarks are characterized by the number of the s quarks they contain, $M_0 \leq M_s \leq M_{ss}$; the energy gap between M_0 and M_{ss} is about $(0.05 - 0.15)\,\text{GeV}$. In this article, we study the interpolating currents which contains zero and two $\bar{s}$ quarks for simplicity.

Lattice QCD calculations for the light flavors indicate that the strong attraction in the scalar diquark channels favors the formation of good diquarks, the weaker attraction (the quark-quark correlation is rather weak) in the axial-vector diquark channels maybe form bad diquarks, the energy gap between the axial-vector and scalar diquarks is about $\frac{2}{3}$ of the Δ-nucleon mass splitting, i.e. $\approx 0.2\,\text{GeV}$ [33, 34], which is expected from the hypersplitting color-spin interaction $\frac{1}{m_i m_j}\vec{T}_i \cdot \vec{T}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$ [31]. The coupled rainbow Dyson-Schwinger equation and ladder Bethe-Salpeter equation also indicate such an energy hierarchy [35]. Comparing with the spin independent term $\vec{T}_i \cdot \vec{T}_j$, the contribution from the hypersplitting color-spin interaction $\frac{1}{m_i m_j}\vec{T}_i \cdot \vec{T}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$ is greatly suppressed by the inverse constituent quark masses. It is possible to form axial-vector diquark states, although the hypersplitting color-spin interaction $\frac{1}{m_i m_j}\vec{T}_i \cdot \vec{T}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$ is repulsive in this channel. If we take the scalar light diquark states as the basic constituents, additional relative P-waves are needed to obtain the correct zero spin. In the conventional quark models, additional P-wave excitation costs about $0.5\,\text{GeV}$, the ground states should be constructed with the axial-vector antidiquark states.

The article is arranged as follows: we derive the QCD sum rules for the scalar doubly charmed and doubly bottom tetraquark states Z in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 QCD sum rules for the scalar tetraquark states Z

In the following, we write down the two-point correlation functions $\Pi(p)$ in the QCD sum rules,

$$
\Pi(p) = i \int d^4 x\, e^{ip\cdot x}\langle 0|T\left[J(x)J^\dagger(0)\right]|0\rangle\,,
\tag{5}
$$

where the $J(x)$ denotes the interpolating currents $J_{qq}(x)$ and $J_{ss}(x)$.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation [4, 5]. After isolating the ground state contribution from the pole

term of the Z, we get the following result,

$$\Pi(p) = \frac{\lambda_Z^2}{M_Z^2 - p^2} + \cdots , \qquad (6)$$

where the pole residue (or coupling) λ_Z is defined by

$$\lambda_Z = \langle 0|J(0)|Z(p)\rangle . \qquad (7)$$

After performing the standard procedure of the QCD sum rules, we obtain the following two sum rules:

$$\lambda_Z^2 e^{-\frac{M_Z^2}{M^2}} = \int_{\Delta_Z}^{s_Z^0} ds \rho_Z(s) e^{-\frac{s}{M^2}} , \qquad (8)$$

the explicit expressions of the spectral densities $\rho_Z(s)$ are presented in the appendix, the s_Z^0 is the continuum threshold parameter and the M^2 is the Borel parameter. We can obtain two sum rules in the $cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ channels with a simple replacement $m_s \to m_q$, $\langle \bar{s}s \rangle \to \langle \bar{q}q \rangle$ and $\langle \bar{s}g_s\sigma Gs \rangle \to \langle \bar{q}g_s\sigma Gq \rangle$.

We carry out the operator product expansion to the vacuum condensates adding up to dimension-10. In calculation, we take vacuum saturation for the high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in large N_c limit. In reality, $N_c = 3$, some ambiguities may come from the vacuum saturation assumption.

We take into account the contributions from the quark condensates, mixed condensates, and neglect the contributions from the gluon condensate. The gluon condensate $\langle \frac{\alpha_s GG}{\pi} \rangle$ is of higher order in α_s, and its contributions are suppressed by very large denominators and would not play any significant role for the light tetraquark states [36, 37], the heavy tetraquark state [6] and the heavy molecular states [38, 39].

In the special case of the $Y(4660)$ (as a $\psi' f_0(980)$ bound state) and its pseudoscalar partner $\eta_c' f_0(980)$, the contributions from the gluon condensate $\langle \frac{\alpha_s GG}{\pi} \rangle$ are rather large [40, 41]. If we take a simple replacement $\bar{s}(x)s(x) \to \langle \bar{s}s \rangle$ and $[\bar{u}(x)u(x) + \bar{d}(x)d(x)] \to 2\langle \bar{q}q \rangle$ in the interpolating currents, the standard heavy quark currents $Q(x)\gamma_\mu Q(x)$ and $Q(x)i\gamma_5 Q(x)$ are obtained, where the gluon condensate $\langle \frac{\alpha_s GG}{\pi} \rangle$ plays an important rule in the QCD sum rules [4]. The interpolating currents constructed from the diquark-antiquark pairs do not have such feature.

We also neglect the terms proportional to the m_u and m_d, their contributions are of minor importance due to the small values of the u and d quark masses.

Differentiating the Eq.(8) with respect to $\frac{1}{M^2}$, then eliminate the pole residues λ_Z, we can obtain the sum rules for the masses of the Z,

$$M_Z^2 = \frac{\int_{\Delta_Z}^{s_Z^0} ds \frac{d}{d(-1/M^2)} \rho_Z(s) e^{-\frac{s}{M^2}}}{\int_{\Delta_Z}^{s_Z^0} ds \rho_Z(s) e^{-\frac{s}{M^2}}} . \qquad (9)$$

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01\,\text{GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2)\langle \bar{q}q \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2\langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.2)\,\text{GeV}^2$,

$m_s = (0.14 \pm 0.01)\,\text{GeV}$, $m_c = (1.35 \pm 0.10)\,\text{GeV}$ and $m_b = (4.8 \pm 0.1)\,\text{GeV}$ at the energy scale $\mu = 1\,\text{GeV}$ [4, 5, 42].

The Q-quark masses appearing in the perturbative terms are usually taken to be the pole masses in the QCD sum rules, while the choice of the m_Q in the leading-order coefficients of the higher-dimensional terms is arbitrary [43, 44]. The $\overline{MS}$ mass $m_c(m_c^2)$ relates with the pole mass $\hat{m}_c$ through the relation $m_c(m_c^2) = \hat{m}_c \left[1 + \frac{C_F \alpha_s(m_c^2)}{\pi} + \cdots\right]^{-1}$. In this article, we take the approximation $m_c(m_c^2) \approx \hat{m}_c$ without the α_s corrections for consistency. The value listed in the Particle Data Group is $m_c(m_c^2) = 1.27^{+0.07}_{-0.11}\,\text{GeV}$ [45], it is reasonable to take $\hat{m}_c = m_c(1\,\text{GeV}^2) = (1.35 \pm 0.10)\,\text{GeV}$. For the b quark, the $\overline{MS}$ mass $m_b(m_b^2) = 4.20^{+0.17}_{-0.07}\,\text{GeV}$ [45], the gap between the energy scale $\mu = 4.2\,\text{GeV}$ and $1\,\text{GeV}$ is rather large, the approximation $\hat{m}_b \approx m_b(m_b^2) \approx m_b(1\,\text{GeV}^2)$ seems rather crude. It would be better to understand the quark masses m_c and m_b we take at the energy scale $\mu^2 = 1\,\text{GeV}^2$ as the effective quark masses (or just the mass parameters). Our previous works on the mass spectrum of the heavy and doubly heavy baryon states indicate such parameters can lead to satisfactory results [27, 28, 29, 46, 47].

In calculation, we also neglect the contributions from the perturbative corrections. Those perturbative corrections can be taken into account in the leading logarithmic approximations through anomalous dimension factors. After the Borel transform, the effects of those corrections are to multiply each term on the operator product expansion side by the factor, $\left[\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right]^{2\Gamma_J - \Gamma_{\mathcal{O}_n}}$, where the Γ_J is the anomalous dimension of the interpolating current $J(x)$ and the $\Gamma_{\mathcal{O}_n}$ is the anomalous dimension of the local operator $\mathcal{O}_n(0)$. We carry out the operator product expansion at a special energy scale $\mu^2 = 1\,\text{GeV}^2$, and set the factor $\left[\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right]^{2\Gamma_J - \Gamma_{\mathcal{O}_n}} \approx 1$, such an approximation maybe result in some scale dependence and weaken the prediction ability. In this article, we study the scalar doubly charmed and doubly bottom tetraquark states systemically, the predictions are still robust as we take the analogous criteria in those sum rules.

In the conventional QCD sum rules [4, 5], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter M^2 and threshold parameter s_0. We impose the two criteria on the scalar doubly heavy tetraquark states to choose the Borel parameter M^2 and threshold parameter s_0.

The vacuum condensates of the high dimension play an important role in choosing the Borel parameter M^2. The condensate of the highest dimension $\langle \bar{s} g_s \sigma G s \rangle^2$ is counted as $\mathcal{O}(\frac{m_Q^2}{M^2})$, $\mathcal{O}(\frac{m_Q^4}{M^4})$ or $\mathcal{O}(\frac{m_Q^6}{M^6})$, and the corresponding contributions are greatly enhanced at small M^2, and result in rather bad convergent behavior in the operator product expansion, we have to choose large Borel parameter M^2. We insist on taking into account the high dimensional vacuum condensates, as the interpolating current consists of a (heavy)diquark-(light)antidiquark pair, one of the highest dimensional vacuum condensates is $\langle \bar{s} s \rangle^2 \langle \frac{\alpha_s GG}{\pi} \rangle$, we have to take into account the condensate $\langle \bar{s} g_s \sigma G s \rangle^2$ for consistence.

The contributions from the high dimension vacuum condensates in the operator product expansion are shown in Fig.1, where (and thereafter) we use the $\langle \bar{s} s \rangle$ to denote the quark condensates $\langle \bar{q} q \rangle$, $\langle \bar{s} s \rangle$ and the $\langle \bar{s} g_s \sigma G s \rangle$ to denote the mixed condensates $\langle \bar{q} g_s \sigma G q \rangle$, $\langle \bar{s} g_s \sigma G s \rangle$. From the figures, we can see that the contributions from the high dimension condensates are very large and change quickly with variation of the Borel parameter at the values $M^2 \leq 2.6\,\text{GeV}^2$ and $M^2 \leq 7.2\,\text{GeV}^2$ in the doubly charmed and doubly bot-

tom channels respectively, such an unstable behavior cannot lead to stable sum rules, our numerical results confirm this conjecture, see Fig.3.

At the values $M^2 \geq 2.6 \, \text{GeV}^2$ and $s_0 \geq 25 \, \text{GeV}^2$, $24 \, \text{GeV}^2$, the contributions from the $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle$ term are less than 9%, 23% in the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$ respectively; the contributions from the vacuum condensate of the highest dimension $\langle \bar{s}g_s\sigma Gs \rangle^2$ are less than 4%, 5% in the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$ respectively; we expect the operator product expansion is convergent in the doubly charmed channels.

At the values $M^2 \geq 7.2 \, \text{GeV}^2$ and $s_0 \geq 140 \, \text{GeV}^2$, $138 \, \text{GeV}^2$, the contributions from the $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle$ term are less than 6%, 16% in the channels $bb\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$ respectively; the contributions from the vacuum condensate of the highest dimension $\langle \bar{s}g_s\sigma Gs \rangle^2$ are less than 5%, 8% in the channels $bb\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$ respectively; we expect the operator product expansion is convergent in the doubly bottom channels.

In this article, we take the uniform Borel parameter M_{min}^2, i.e. $M_{min}^2 \geq 2.6 \, \text{GeV}^2$ and $M_{min}^2 \geq 7.2 \, \text{GeV}^2$ in the doubly charmed and doubly bottom channels respectively.

In Fig.2, we show the contributions from the pole terms with variation of the Borel parameters and the threshold parameters. The pole contributions are larger than (or equal) 50%, 47% at the value $M^2 \leq 3.3 \, \text{GeV}^2$ and $s_0 \geq 25 \, \text{GeV}^2$, $24 \, \text{GeV}^2$ in the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$ respectively, and larger than (or equal) 52%, 50% at the value $M^2 \leq 8.2 \, \text{GeV}^2$ and $s_0 \geq 140 \, \text{GeV}^2$, $138 \, \text{GeV}^2$ in the channels $bb\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$ respectively. Again we take the uniform Borel parameter M_{max}^2, i.e. $M_{max}^2 \leq 3.3 \, \text{GeV}^2$ and $M_{max}^2 \leq 8.2 \, \text{GeV}^2$ in the doubly charmed and doubly bottom channels respectively.

In this article, the threshold parameters are taken as $s_0 = (26 \pm 1) \, \text{GeV}^2$, $(25 \pm 1) \, \text{GeV}^2$, $(142 \pm 2) \, \text{GeV}^2$, $(140 \pm 2) \, \text{GeV}^2$ in the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$ respectively; the Borel parameters are taken as $M^2 = (2.6 - 3.3) \, \text{GeV}^2$ and $(7.2 - 8.2) \, \text{GeV}^2$ in the doubly charmed and doubly bottom channels respectively. In those regions, the pole contributions are about $(50 - 80)\%$, $(47 - 78)\%$, $(52 - 71)\%$, $(50 - 70)\%$ in the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$ respectively; the two criteria of the QCD sum rules are fully satisfied [4, 5].

From Fig.2, we can see that the Borel windows $M_{max}^2 - M_{min}^2$ change with variations of the threshold parameters s_0. In this article, the Borel windows are taken as $0.7 \, \text{GeV}^2$ and $1.0 \, \text{GeV}^2$ in the doubly charmed and doubly bottom channels respectively; they are small enough. If we take larger threshold parameters, the Borel windows are larger and the resulting masses are larger, see Fig.3. In this article, we intend to calculate the possibly lowest masses which are supposed to be the ground state masses by imposing the two criteria of the QCD sum rules.

Taking into account all uncertainties of the relevant parameters, finally we obtain the values of the masses and pole resides of the scalar doubly heavy tetraquark states Z, which are shown in Figs.4-5 and Table 1.

In this article, we calculate the uncertainties δ with the formula

$$\delta = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i} \right)^2 |_{x_i = \bar{x}_i} (x_i - \bar{x}_i)^2} \,, \tag{10}$$

where the f denote the hadron mass M_Z and the pole residue λ_Z, the x_i denote the relevant parameters m_c, m_b, $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$, $\cdots$. As the partial derivatives $\frac{\partial f}{\partial x_i}$ are difficult to carry out analytically, we take the approximation $\left(\frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i)]^2$ in the numerical calculations.
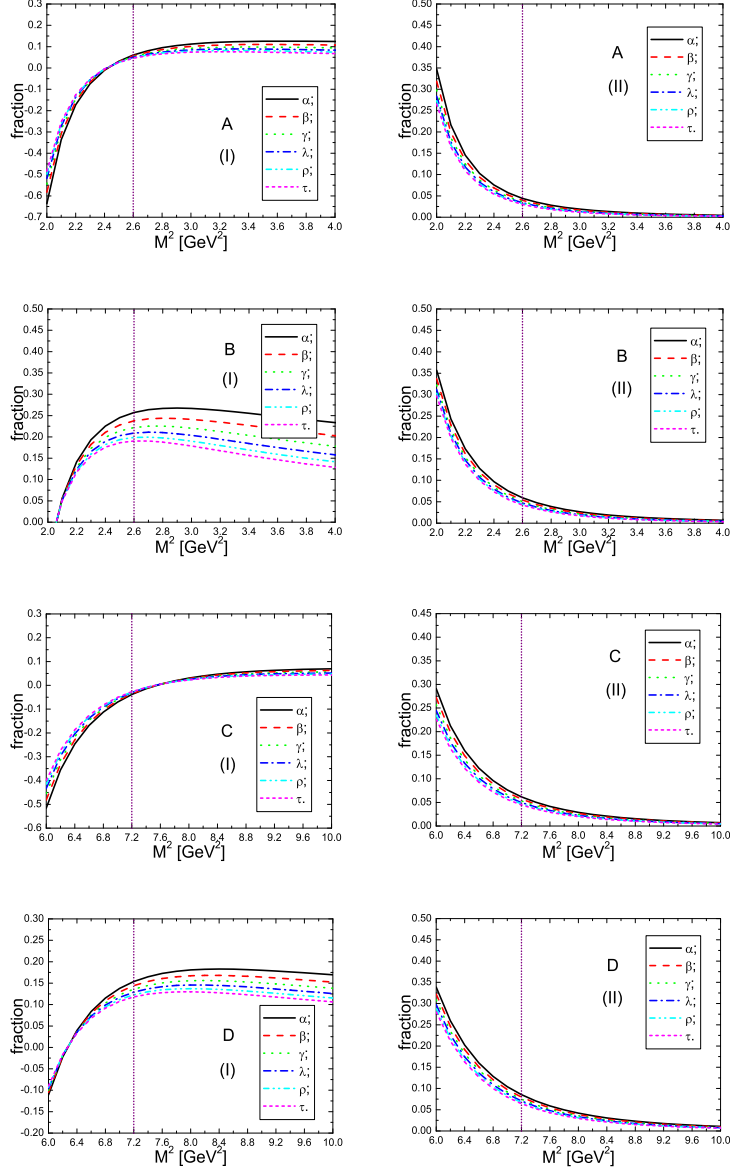
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Figure 1: The contributions from different terms with variation of the Borel parameter M^2 in the operator product expansion. The (I) and (II) denote the contributions from the $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle$ term and the $\langle \bar{s}g_s\sigma Gs \rangle^2$ term respectively. The A, B, C and D denote the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$ and $bb\bar{q}\bar{q}$ respectively. The notations α, β, γ, λ, ρ and τ correspond to the threshold parameters $s_0 = 22\,\mathrm{GeV}^2$, $23\,\mathrm{GeV}^2$, $24\,\mathrm{GeV}^2$, $25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$ and $27\,\mathrm{GeV}^2$ respectively in the doubly charmed channels; while they correspond to the threshold parameters $s_0 = 134\,\mathrm{GeV}^2$, $136\,\mathrm{GeV}^2$, $138\,\mathrm{GeV}^2$, $140\,\mathrm{GeV}^2$, $142\,\mathrm{GeV}^2$ and $144\,\mathrm{GeV}^2$ respectively in the doubly bottom channels.
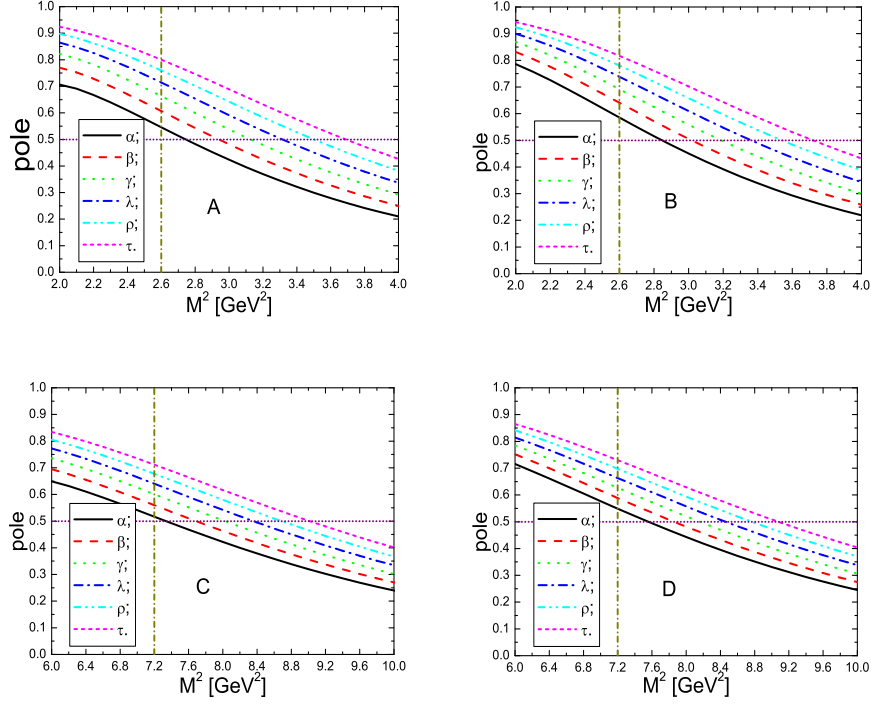
Figure 2: The contributions of the pole terms with variation of the Borel parameter M^2. The A, B, C and D denote the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$ and $bb\bar{q}\bar{q}$ respectively. The notations α, β, γ, λ, ρ and τ correspond to the threshold parameters $s_0 = 22\,\mathrm{GeV}^2$, $23\,\mathrm{GeV}^2$, $24\,\mathrm{GeV}^2$, $25\,\mathrm{GeV}^2$, $26\,\mathrm{GeV}^2$ and $27\,\mathrm{GeV}^2$ respectively in the doubly charmed channels; while they correspond to the threshold parameters $s_0 = 134\,\mathrm{GeV}^2$, $136\,\mathrm{GeV}^2$, $138\,\mathrm{GeV}^2$, $140\,\mathrm{GeV}^2$, $142\,\mathrm{GeV}^2$ and $144\,\mathrm{GeV}^2$ respectively in the doubly bottom channels.

tetraquark states	M_Z	λ_Z
$cc\bar{s}\bar{s}$	4.52 ± 0.18	0.154 ± 0.032
$cc\bar{q}\bar{q}$	4.35 ± 0.16	0.126 ± 0.033
$bb\bar{s}\bar{s}$	11.32 ± 0.18	0.825 ± 0.180
$bb\bar{q}\bar{q}$	11.14 ± 0.16	0.660 ± 0.164

Table 1: The masses and the pole residues of the scalar doubly heavy tetraquark states. The masses are in unit of GeV and the pole residues are in unit of GeV^5.
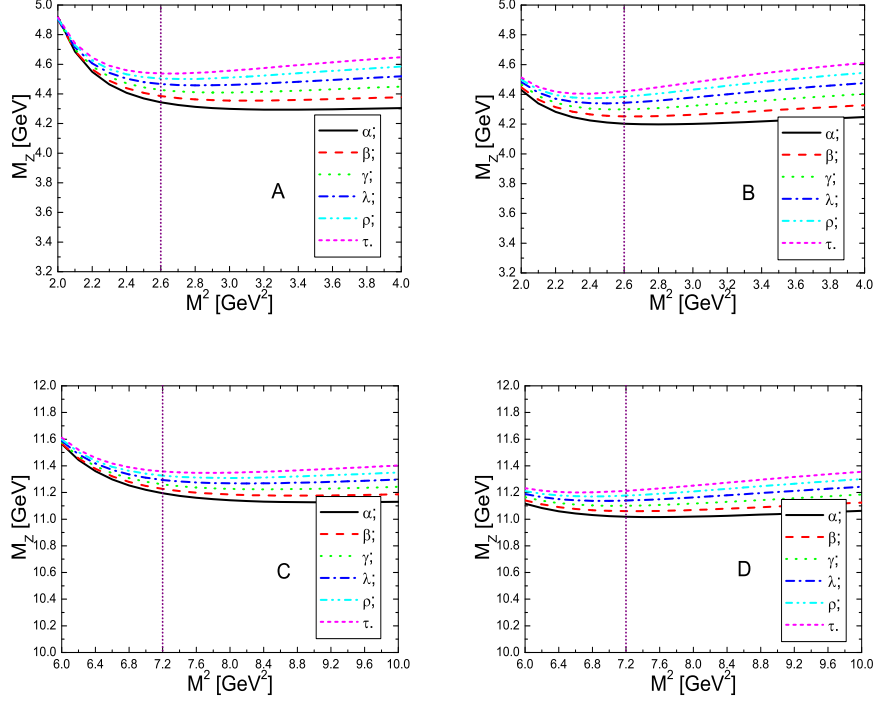
Figure 3: The masses of the scalar doubly heavy tetraquark states with variation of the Borel parameter M^2. The A, B, C and D denote the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$ and $bb\bar{q}\bar{q}$ respectively. The notations α, β, γ, λ, ρ and τ correspond to the threshold parameters $s_0 = 22\,\text{GeV}^2$, $23\,\text{GeV}^2$, $24\,\text{GeV}^2$, $25\,\text{GeV}^2$, $26\,\text{GeV}^2$ and $27\,\text{GeV}^2$ respectively in the doubly charmed channels; while they correspond to the threshold parameters $s_0 = 134\,\text{GeV}^2$, $136\,\text{GeV}^2$, $138\,\text{GeV}^2$, $140\,\text{GeV}^2$, $142\,\text{GeV}^2$ and $144\,\text{GeV}^2$ respectively in the doubly bottom channels.

	This work	[26]	[8]*	[49, 50]*	[51, 52, 53]*
$cc\bar{s}\bar{s}$	4.52 ± 0.18	4.359	4.45 ± 0.16	3.967	3.927
$cc\bar{q}\bar{q}$	4.35 ± 0.16	4.056	4.36 ± 0.18	3.852	3.832
$bb\bar{s}\bar{s}$	11.32 ± 0.18	10.932	11.23 ± 0.16	10.671	10.874
$bb\bar{q}\bar{q}$	11.14 ± 0.16	10.648	11.14 ± 0.19	10.473	10.528

Table 2: The masses of the scalar doubly heavy tetraquark states, the star denotes the corresponding $Qq\bar{Q}\bar{q}'$ type tetraquark states. The masses are in unit of GeV.
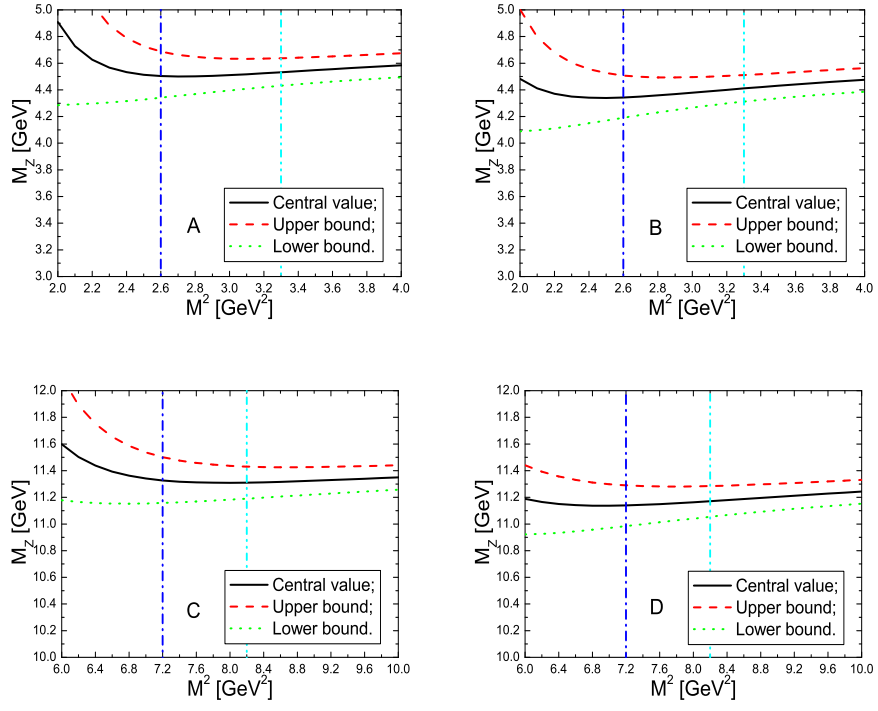
Figure 4: The masses of the scalar doubly heavy tetraquark states with variation of the Borel parameter M^2. The A, B, C and D denote the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$ and $bb\bar{q}\bar{q}$ respectively.
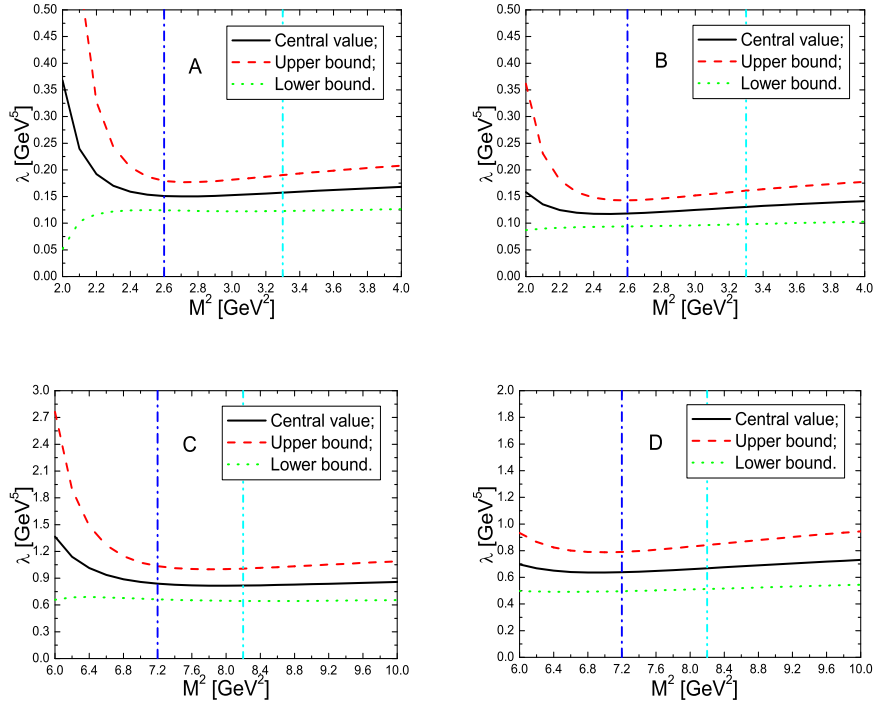
Figure 5: The pole residues of the scalar doubly heavy tetraquark states with variation of the Borel parameter M^2. The A, B, C and D denote the channels $cc\bar{s}\bar{s}$, $cc\bar{q}\bar{q}$, $bb\bar{s}\bar{s}$ and $bb\bar{q}\bar{q}$ respectively.

Naively, we expect the $QQ\bar{q}\bar{q}'$ and $Qq\bar{Q}\bar{q}'$ type tetraquark states have degenerate masses as the color interactions are flavor blinded. However, we cannot obtain a relation between the corresponding interpolating currents by Fierz reordering in the color and Dirac spinor spaces as different flavor structures are concerned. Furthermore, additional contributions from the instanton configurations make the situation more complicated [48]. In Table 2, we also present the values of the $QQ\bar{q}\bar{q}'$ and $Qq\bar{Q}\bar{q}'$ type tetraquark states from the relativistic quark model based on a quasipotential approach in QCD [26, 49, 50], the constituent diquark model plus the spin-spin interactions [51, 52, 53], and the QCD sum rules [8]. The QCD sum rules indicate the $QQ\bar{q}\bar{q}'$ and $Qq\bar{Q}\bar{q}'$ type tetraquark states have almost degenerate masses, while the central values of present predictions are larger than the corresponding ones from other theoretical models about $(0.2 - 0.7)\,\mathrm{GeV}$.

In Refs.[26, 49, 50], Ebert et al take the diquarks as bound states of the two quarks in the color antitriplet channel, and calculate their mass spectrum using a Schrodinger type equation, then take the masses of the diquarks as the basic input parameters, and study the mass spectrum of the heavy tetraquark states as bound states of the diquark-antidiquark system. In Refs.[51, 52, 53], Maiani et al take the diquarks as the basic constituents, examine the rich spectrum of the diquark-antidiquark states with the constituent diquark masses and the spin-spin interactions, and try to accommodate some of the newly observed charmonium-like and bottomonium-like resonances not fitting pure $c\bar{c}$ and $b\bar{b}$ assignment. The predictions depend heavily on the assumption that the light scalar mesons $a_0(980)$ and $f_0(980)$ are tetraquark states, the basic parameters (constituent diquark masses) are estimated thereafter. In the conventional quark models, the constituent quark masses are taken as the basic input parameters, and fitted to reproduce the mass spectra of the well known mesons and baryons. However, the present experimental knowledge about the phenomenological hadronic spectral densities of the tetraquark states is rather vague, the constituent diquark masses and thereafter predictions cannot be confronted with the experimental data.

If kinematically allowed, the scalar doubly heavy tetraquark states Z can decay to the heavy meson pairs with the Okubo-Zweig-Iizuka super-allowed "fall-apart" mechanism, i.e. $QQ\bar{q}\bar{q} \to Q\bar{q}\,Q\bar{q}$ and $QQ\bar{s}\bar{s} \to Q\bar{s}\,Q\bar{s}$. The thresholds for the DD, D_sD_s, D^*D^*, $D_s^*D_s^*$, BB, B_sB_s, B^*B^* and $B_s^*B_s^*$ are about $3.74\,\mathrm{GeV}$, $3.94\,\mathrm{GeV}$, $4.02\,\mathrm{GeV}$, $4.22\,\mathrm{GeV}$, $10.56\,\mathrm{GeV}$, $10.73\,\mathrm{GeV}$, $10.65\,\mathrm{GeV}$ and $10.83\,\mathrm{GeV}$, respectively [45]. From Table 1, we can see that the strong decays $Z_{cc\bar{q}\bar{q}} \to DD$, D^*D^* and $Z_{cc\bar{s}\bar{s}} \to D_sD_s$, $D_s^*D_s^*$ are kinematically allowed, the phase spaces are rather large; while the corresponding decays for the doubly bottom tetraquark states are kinematically forbidden.

The doubly heavy tetraquark states can also decay to the baryon pairs with creation of the $q\bar{q}$ or $s\bar{s}$ pairs from the QCD vacuum, $QQ\bar{q}\bar{q} \to QQq' + \bar{q}'\bar{q}\bar{q}$, $QQs + \bar{s}\bar{q}\bar{q}$, $QQ\bar{s}\bar{s} \to QQq + \bar{q}\bar{s}\bar{s}$, $QQs + \bar{s}\bar{s}\bar{s}$. However, the strong decays to $\Xi_{QQ}\bar{p}$, $\Xi_{QQ}^*\bar{\Delta}$, $\Omega_{QQ}\bar{\Sigma}$, $\Omega_{QQ}^*\bar{\Sigma}^*$, $\Xi_{QQ}\bar{\Xi}'$, $\Xi_{QQ}^*\bar{\Xi}^*$, $\Omega_{QQ}\bar{\Omega}$ and $\Omega_{QQ}^*\bar{\Omega}^*$ are kinematically forbidden or greatly suppressed. The scalar doubly charmed tetraquark states maybe have large widths, while the scalar doubly bottom tetraquark states maybe have very small widths.

In 2002, the SELEX collaboration reported the first observation of a signal for the doubly charm baryon state Ξ_{cc}^+ in the charged decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ [54], and confirmed later by the same collaboration in the decay mode $\Xi_{cc}^+ \to pD^+K^-$ with measured mass $M_\Xi = 3518.9 \pm 0.9\,\mathrm{MeV}$ [55]. No other doubly heavy baryon states are observed. We use the masses of the $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ doubly heavy baryon states Ξ_{QQ}, Ξ_{QQ}^*, Ω_{QQ}, Ω_{QQ}^* from

the QCD sum rules [27, 28, 29].

The LHCb is a dedicated b and c-physics precision experiment at the LHC (large hadron collider). The LHC will be the world's most copious source of the b hadrons, and a complete spectrum of the b hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14\,\text{TeV}$, the $b\bar{b}$ cross section is expected to be $\sim 500\mu b$ producing 10^{12} $b\bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ [56]. The scalar doubly heavy tetraquark states predicted in the present work may be observed at the LHCb, if they exist indeed.

4   Conclusion

In this article, we study the mass spectrum of the scalar doubly charmed and doubly bottom tetraquark states with the QCD sum rules in a systematic way. The mass spectrum are calculated by imposing the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. The present predictions can be confronted with the experimental data in the future at the LHCb.

Appendix

The spectral densities $\rho_Z(s)$ at the level of the quark-gluon degrees of freedom:

$$\rho_Z(s) = \frac{1}{32\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha\beta(1-\alpha-\beta)^3 (s-\widetilde{m}_Q^2)^2 (7s^2 - 6s\widetilde{m}_Q^2 + \widetilde{m}_Q^4)$$

$$+ \frac{m_Q^2}{32\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1-\alpha-\beta)^2 (s-\widetilde{m}_Q^2)^3$$

$$+ \frac{m_s\langle\bar{s}s\rangle}{\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha\beta(1-\alpha-\beta)(10s^2 - 12s\widetilde{m}_Q^2 + 3\widetilde{m}_Q^4)$$

$$- \frac{m_s\langle\bar{s}s\rangle}{\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha\beta(s-\widetilde{m}_Q^2)(2s-\widetilde{m}_Q^2)$$

$$- \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha\beta \left[2s - \widetilde{m}_Q^2 + \frac{s^2}{6}\delta(s-\widetilde{m}_Q^2) \right]$$

$$- \frac{3m_s m_Q^2\langle\bar{s}s\rangle}{2\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s-\widetilde{m}_Q^2)$$

$$+ \frac{4m_Q^2\langle\bar{s}s\rangle^2}{3\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha + \frac{2\langle\bar{s}s\rangle^2}{3\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1-\alpha)(3s-2\widetilde{\widetilde{m}}_Q^2)$$

$$+ \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{4\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1-\alpha)(3s-2\widetilde{\widetilde{m}}_Q^2)$$

$$+ \frac{5m_s m_Q^2\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha$$

$$- \frac{2m_Q^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{3\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[1 + \frac{s}{M^2} \right] \delta(s-\widetilde{\widetilde{m}}_Q^2)$$

$$- \frac{\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1-\alpha) \left\{ 2 + \left[\frac{4s}{3} + \frac{s^2}{3M^2} \right] \delta(s-\widetilde{\widetilde{m}}_Q^2) \right\}$$

$$+ \frac{m_Q^2\langle\bar{s}g_s\sigma Gs\rangle^2}{12\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha s^2 \delta(s-\widetilde{\widetilde{m}}_Q^2)$$

$$+ \frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{4\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1-\alpha) \left[1 + \frac{s}{M^2} + \frac{s^2}{2M^4} + \frac{s^3}{6M^6} \right] \delta(s-\widetilde{\widetilde{m}}_Q^2), \quad (11)$$

where $\alpha_f = \frac{1+\sqrt{1-4m_Q^2/s}}{2}$, $\alpha_i = \frac{1-\sqrt{1-4m_Q^2/s}}{2}$, $\beta_i = \frac{\alpha m_Q^2}{\alpha s - m_Q^2}$, $\widetilde{m}_Q^2 = \frac{(\alpha+\beta)m_Q^2}{\alpha\beta}$, $\widetilde{\widetilde{m}}_Q^2 = \frac{m_Q^2}{\alpha(1-\alpha)}$, and $\Delta_Z = 4(m_Q + m_s)^2$.

Acknowledgements

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